

Finite element study for determination of material's creep parameters from small punch test[†]

Jianjun Chen¹, Young Wha Ma¹ and Kee Bong Yoon^{2,*}

¹Research Associates, Department of Mechanical Engineering, Chung Ang University, 221 Huksuk Dongjak, Seoul, 156-756, Korea

²Department of Mechanical Engineering, Chung Ang University, 221 Huksuk Dongjak, Seoul, 156-756, Korea

(Manuscript Received November 18, 2008; Revised July 15, 2009; Accepted January 28, 2010)

Abstract

Compared with the conventional tensile creep test, it is much more difficult to obtain the creep properties of a material by the small punch creep test due to the complex deformation response and stress distribution in the miniature specimen of the material. Although creep behavior has been investigated by the small punch test, most studies have been limited to a specimen geometry and therefore, cannot be extended to other conditions conveniently. In this study, a new developed analysis routine is presented to derive the creep parameters of a material using data obtained from the small punch creep test. With the aid of the finite element method, the displacement and the displacement rate of the small punch are obtained for different load levels. The relationship between the stress and creep strain of the specimen and the applied load and the punch displacement is obtained by a dimensional analysis and the membrane stretching model. The creep properties obtained from small punch tests and the conventional creep tests are also compared. The values of the creep properties between the two types of tests agree well with each other within an acceptable accuracy range. This indicates that it is possible to obtain the creep parameters of a material from the small punch creep test instead of the conventional creep test by the analysis routines proposed in this study. Some suggestions for data reduction of the small punch creep tests are also presented to obtain more accurate material creep parameters.

Keywords: Small punch test; Creep; Finite element method; Dimensional analysis; Membrane stretching

1. Introduction

Tensile creep test is a standard test technique used to obtain material's creep properties. However, it has some limitations in determining the creep properties of an in-service component. Because the conventional creep test needs considerable amounts of the test material from an operating component, the whole plant having the component may have to be shutdown. To overcome the above shortcoming, a novel small punch testing method has been developed since 1980s to determine the creep properties of a material. This method uses miniature specimens of the test material [1]. The small punch creep test requires less amount of material removal and avoids additional repair processes to the component. Because this test requires small test specimens, it is generally classified as a non-destructive method for obtaining the property of an operating component without requiring the sacrifice of the component's structural integrity.

The small punch test has been widely used to characterize material properties, such as strength, flexibility and fracture toughness, at room temperature [2]. Recently, several significant improvements have been made on small punch creep test to determine creep behavior [3-5]. A numerical model has been progressed for the small punch testing under the constant deflection rate condition [6]. Yoon et al also reported that it may be possible to assess creep constants from time-displacement curves obtained from a small punch creep test [7, 8]. Their studies implied that the accurate creep properties obtained from the small punch test are close to those obtained by the tensile creep tests with standard size specimens [9]. However, the currently applied relationships between the data of the conventional creep test and the small punch creep test are purely empirical. Most of the research results are only valid for a specific material or a given geometry. The data derived for specimens of different shapes are highly dispersed. Although the European and US research groups have standardized the dimensions and the test conditions of the small punch test at high temperatures [10], they have yet to find a general way to directly link the creep properties obtain from the small punch test and the tensile creep test. As such, the

[†] This paper was recommended for publication in revised form by Associate Editor Youngseog Lee

*Corresponding author. Tel.: +82 2 820 5328, Fax.: +82 2 812 6474

E-mail address: kbyoon@cau.ac.kr

© KSME & Springer 2010

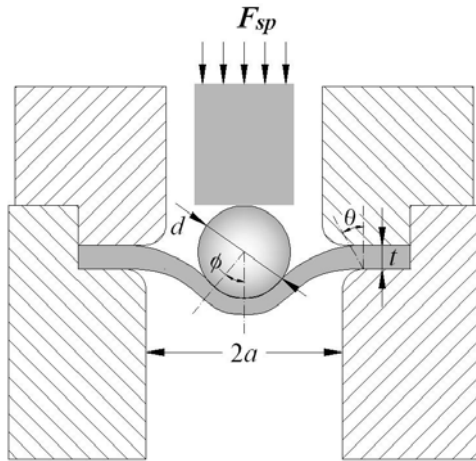


Fig. 1. Schematic of the small punch creep test apparatus.

aim of this work is to propose an analysis routine to determine the creep properties, such as creep coefficient and creep exponent, of a Norton’ power law creeping material from small punch tests. The derived creep properties are then compared with those obtained from tensile creep tensile tests to examine the difference between these two results. Using the finite element method a detailed study is performed to verify and better understand the small punch test for deriving a material’s creep properties.

2. Theoretical analysis

The small punch test technique is based on the deformation measurement of a small disk-shaped specimen with an applied central force. The schematic illustration of a small punch test is shown in Fig. 1. To obtain the creep property values accurately, a vacuum furnace is always recommended to mitigate the oxidation effect on the thin test specimen. Assuming the material obeys the Norton’s power creep law, creep strain rate under the steady state is defined as

$$\dot{\epsilon} = B\sigma^n \tag{1}$$

where, B is the creep coefficient and n is the creep exponent. When a rigid punch is pressed down into a power-law creeping specimen, the impression load can be expressed as a function of several variables which include the displacement of the punch head δ , the velocity of the punch head $\dot{\delta}(=d\delta/dt)$, the creep properties of the specimen material, the geometric parameters of the punch in the contact region and the shape parameters determining the size of the material’s response area. From the PI-theorem of dimensional analysis [11], the indentation load F_{sp} can be expressed in the following form

$$F_{sp} = \left(B \frac{\dot{\delta}}{\delta} \right)^{\frac{1}{n}} \delta^2 f \left[n, \left(\frac{d}{\delta} \right)^{c_1}, \left(\frac{t}{\delta} \right)^{c_2}, \left(\frac{a}{\delta} \right)^{c_3} \right] \tag{2}$$

where d is the diameter of the punch head, t is the specimen thickness and a is the inner radius of the lower die. The real parameters, c_1 , c_2 and c_3 are the regression constants and f represents the dimensionless function of each non-dimensional parameters. Under the two different loads F_{sp1} and F_{sp2} , the punch head may have two different displacement rates $\dot{\delta}_1$ and $\dot{\delta}_2$ at a given indentation depth $\bar{\delta}$, which can be expressed as

$$F_{sp1} = \left(B \frac{\dot{\delta}_1}{\bar{\delta}} \right)^{\frac{1}{n}} \bar{\delta}^2 f_1 \left[n, \left(\frac{d}{\bar{\delta}} \right)^{c_1}, \left(\frac{t}{\bar{\delta}} \right)^{c_2}, \left(\frac{a}{\bar{\delta}} \right)^{c_3} \right] \tag{3}$$

$$F_{sp2} = \left(B \frac{\dot{\delta}_2}{\bar{\delta}} \right)^{\frac{1}{n}} \bar{\delta}^2 f_2 \left[n, \left(\frac{d}{\bar{\delta}} \right)^{c_1}, \left(\frac{t}{\bar{\delta}} \right)^{c_2}, \left(\frac{a}{\bar{\delta}} \right)^{c_3} \right] \tag{4}$$

Noticing that the effective geometric parameters corresponding to the two different displacement rates are the same for the spherical punch head, i.e. $f_1 = f_2$, the creep index n can thus be easily obtained from Eqs. (3) and (4) as

$$n = \frac{\log \dot{\delta}_1 - \log \dot{\delta}_2}{\log F_{sp1} - \log F_{sp2}} \tag{5}$$

Above equations were also obtained from the dimensional analysis performed by Cheng and Cheng using a self-similar indenter and a power law creep material [12]. Eq. (5) indicates that the creep exponent n obtained from the small punch creep test by using load-displacement velocity data should be identical to that from the conventional tensile creep test. Such finding agrees with the conclusions in [13] and [14] as well.

The relationship between the deflection and membrane strain under the spherical punch was firstly studied by Chakrabarty [15]. In his paper, the disc specimen was assumed to be deformed under membrane stresses since the thickness of the disc was small in comparison to the radius of the punch head. Chakrabarty pointed out that the maximum equivalent strain inside the specimen can be approximated as

$$\epsilon \cong 2 \ln \frac{2(1 + \cos \theta)}{(1 + \cos \phi)^2} \tag{6}$$

The definitions of θ and ϕ are illustrated in Fig. 1. When the displacement of the punch head increases, the contact surface will become large and the membrane stress will vary with time. Chakrabarty simplified such a problem and gave an expression for the central deflection of the disc (identical to the displacement of the punch head) as

$$\delta \cong a \sin \theta \ln \frac{\tan(\phi/2)}{\tan(\theta/2)} + \frac{d}{2}(1 - \cos \phi) \tag{7}$$

Note that the geometry relationship between the θ and ϕ is given by Eq. (8)

$$\sin \theta = \frac{d}{2a} \sin^2 \varphi \tag{8}$$

Substitution of Eq. (8) into Eq. (6) and Eq. (7) gives

$$\varepsilon \cong 2 \ln \frac{2 + \sqrt{4 - \frac{d^2 \sin^4 \varphi}{a^2}}}{(1 + \cos \varphi)^2} \tag{9}$$

and

$$\delta = \frac{d}{2} \left\{ 1 - \cos \varphi + \ln \left[\cot \left(\frac{1}{2} \operatorname{arccsc} \frac{2a \csc^2 \varphi}{d} \right) \tan \frac{\varphi}{2} \right] \sin^2 \varphi \right\} \tag{10}$$

Assuming φ to be changed from 0° to 90° , we can obtain the implicit functions between variables δ and ε according to Eq. (9) and Eq. (10). A simplified polynomial function can be further regressed to directly link these two variables.

Using the relationship based on the membrane stretching assumption, an empirical correlation between the applied load and the membrane stress can be obtained as [15]

$$\frac{F_{sp}}{\sigma} \cong \pi t d \sin^2 \varphi \tag{11}$$

Substituting Eq. (11) and the regressed formulae between δ and ε into Eq. (1) the creep parameter B can be finally determined.

3. Finite element simulation

To obtain the relationship between the load and the deflection displacement, a numerical simulation study was performed. The small punch creep test can be simplified as an axi-symmetric model in finite element analysis. In the analysis, the specimen material was assumed to be Gr91 steel at 565°C and its mechanical properties were obtained from the conventional tensile creep test as given in Table 1.

The thickness t of the small punch specimen was assumed to be 0.5 mm and the diameter of the punch head d was 2mm. The specimen was clamped around its circumference with radius $a = 2$ mm, as shown in Fig. 1. The finite element software ABAQUS [16] was used to model and calculate the time-dependent performance of the small punch test. The small punch head was assumed to be rigid during the punching process. The contact between the specimen and the punch head was modeled by surface to surface contact elements and the friction coefficient was set to 0.2 in the analysis. The two-dimensional axi-symmetric finite element mesh is illustrated in Fig. 2, which contains 1630 elements and 1754 nodes. The reduced integration element type CAX4R is chosen to prevent the shear and volume locking due to severe shear and torsion effects, which tend to occur in a contact simulation.

Table 1. Mechanical properties of Gr91 steel at 565°C .

Elastic Modulus (GPa)	Yield Strength (MPa)	Tensile Strength (MPa)	B ($\text{MPa}^{-n}\text{h}^{-1}$)	n
145.6	374.8	421.2	1.35×10^{-34}	12.75

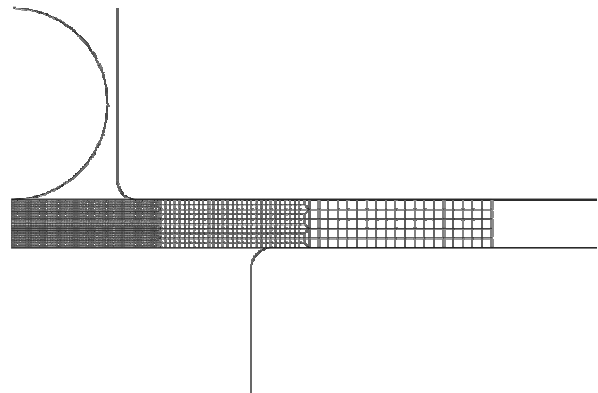


Fig. 2. Axi-symmetric finite element mesh for the small punch creep test analysis.

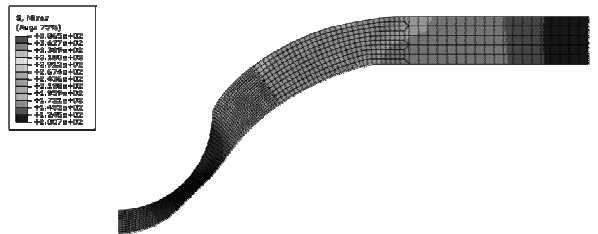


Fig. 3. Deformation and stress distribution in the small punch specimen.

The finite element simulations for the small punch creep test were performed for four different constant loads of 500N, 450N, 400N and 300N. The contour of the equivalent stress can be seen in Fig. 3. The maximum stress was not located at the center of the specimen but it deviated a distance from the center point. The weakest location did not almost change after the specimen entered the steady state creep condition. The thickness at the weakest point reduced gradually with time and the specimen ultimately ruptured at this point. This result has been verified by many experiments [6, 13].

4. Results and discussion

4.1 Determination of creep exponent

On the basis of the finite element analysis, the displacement of the punch under $F_{sp} = 500\text{N}$ is plotted in Fig. 4. Similar to the conventional tensile creep test curve, the small punch creep curve also exhibits clear primary, secondary and tertiary creep states. Since there is no essential difference in the creep mechanism between the tensile test and the small punch test, it is possible to derive the appropriate creep properties from the small punch test data. However, during the small punch test,

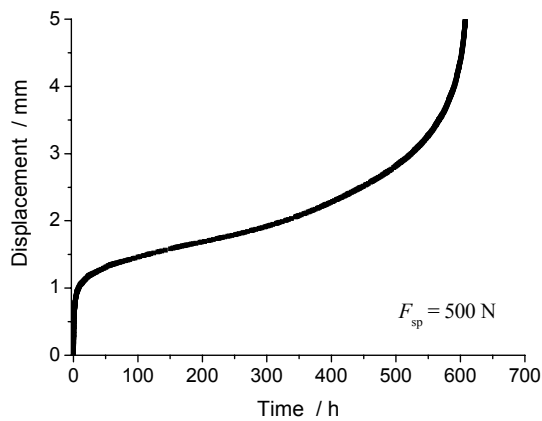


Fig. 4. Small punch creep displacement curve when the applied load is 500N.

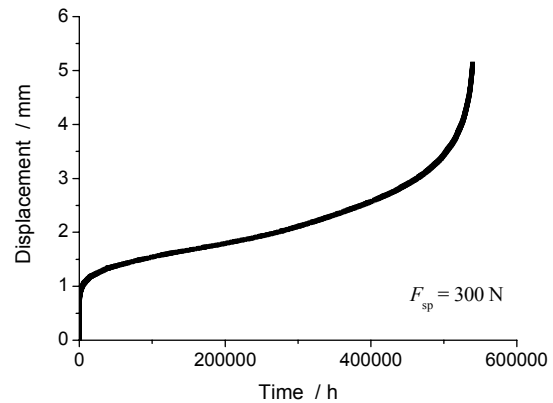


Fig. 6. Small punch creep displacement curve when the applied load is 300N.

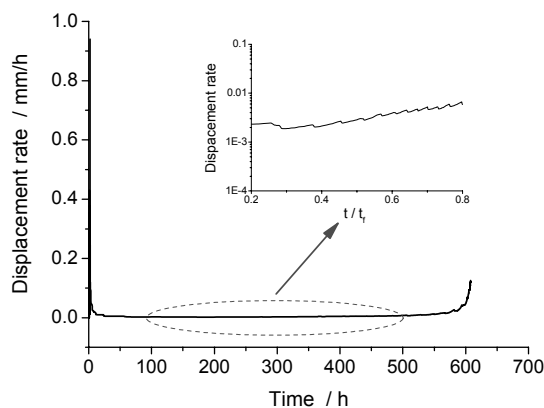


Fig. 5. Small punch creep displacement rate curve.

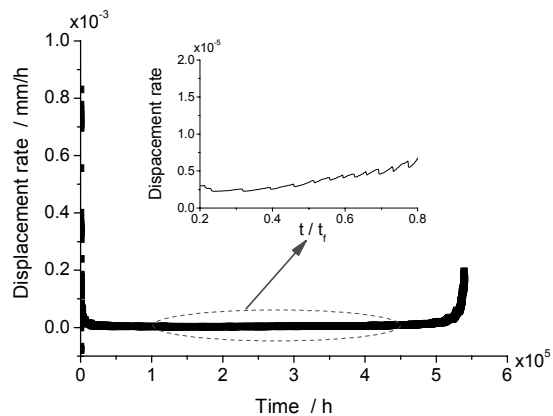


Fig. 7. Small punch creep displacement rate curve when the applied load is 300N.

only punch displacement (unit: mm) can be measured at different times under a certain applied load (unit: N). Because of the dimensional difference between tensile creep specimen and small punch test specimen, the data manipulation method which is widely used in the analysis of tensile creep test data cannot be adopted to treat the small punch test data directly. The simplest idea is to convert the displacement and the applied load data into the strain and stress values of the specimen. But such conversion formulae must be influenced by many factors and appear to be quite different with each other, some being even contrary to other works. Fortunately, from the theoretical derivation in section 2 we showed that the creep exponent n can be derived from a log-linear regression according to Eq. (5) and should be identical to that obtained from the conventional tensile creep test data.

The displacement rate of the small punch test can be obtained by differentiating the displacement data with time, which is plotted in Fig. 5. In data reduction of the actual creep test, the measured displacement rate values are quite small even in the steady state. Hence, selecting the minimum values of the displacement rate as the steady state data may not be a good choice because the environment noise and apparatus instability sometimes cause unusual minimum displacement

rate. Using such incorrect minimum data may lead to inaccurate results. More importantly, the creep exponent can be derived from Eq. (5) only under the same given displacement value of $\bar{\delta}$. However, the point on a creep curve, where the fixed displacement value is $\bar{\delta}$ does not stand for the minimum displacement rate in general. So a reliable method should be established for selecting a series of data containing the minimum creep displacement rate values and being regarded as data in the steady state creep stage.

In the following analysis, the data between the 0.2 life time to 0.8 life time are selected as the steady state condition values. This selected bound will cover most of the possible creep steady state data as much as it can. It will be seen from the following analysis that accurate creep parameters can be obtained from the above life time bounds even if there are some data which do not belong to the secondary creep state. In Fig. 5, it was observed that the displacement rates undulated within a narrow fluctuation range. This was expected even in the actual tests. The development of the displacement and the displacement rate under the load of $F_{sp} = 300N$ are shown in Fig. 6 and Fig. 7, respectively. They have similar curve shapes as those shown in Fig. 4 and Fig. 5 except a longer life time and the lower displacement rate due to the lower stress level.

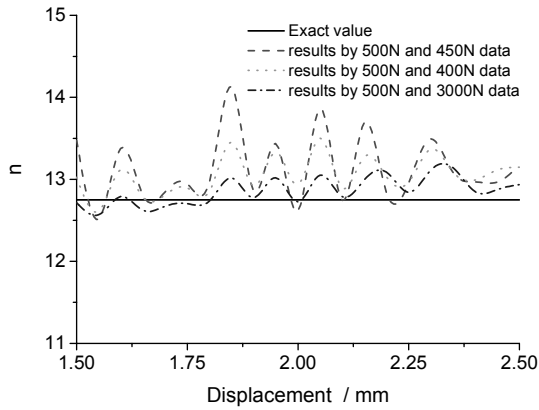


Fig. 8. Creep exponent(n) obtained from different load values.

According to Eq. (5), the creep exponent value n can be obtained from any two different loads. However, in an actual calculation, the derived property data is also affected by the boundary condition and random environmental factors. In the following analyses, the creep exponent n is calculated from three different sets of loads and the results are shown in Fig. 8. As expected, in every case the calculated n value is close to its real value when the given displacement $\bar{\delta}$ is changed from 1.5 to 2.5 mm. But the value of n obtained from the load levels of 500N and 450N showed a larger deviation than that obtained from other load conditions. The n value obtained from the loads of 500N and 300N is closest to its exact value. This means that although any two different loads may yield the same creep exponent theoretically, a larger load difference is advantageous for obtaining a more accurate creep exponent. This is partially because the close load level will result in a smaller denominator in Eq. (5) and magnify the irrelevant influencing factors easily to produce a larger creep exponent value. If the two loads levels are greatly different, the degree of influence of the other impact factors will reduce and the final derived n becomes more precise.

The relative error of the creep exponent n is shown in Fig. 9. Even the largest error from two close load levels (500N and 450N) is no more than 10%. For the 500N and 300N load condition, the relative error value is much less than 3% on the whole. The average creep exponent value from different load levels is listed in Table 2. All the relative errors are less than 3%. Such accuracy grade can meet most of engineering requirements pertaining to the determination of a material's creep parameters. However, it should be noted that a lower load level needs a longer test time, so the cost of an accurate result (for a larger load difference) is more experiment time. The compromise between accurate property values and test time or test cost should be considered in actual experiments.

4.2 Determination of the creep coefficient

To evaluate the creep coefficient value B from the small punch test, the relationship between the small punch displacement and the strain inside the specimen should be estab-

Table 2. Average creep exponent value n obtained from various load levels.

	Average value for $\bar{\delta} = 1.5\text{-}2.5$ mm		
Load level	500N/450N	500N/400N	500N/300N
n	13.13	13.05	12.85
Error (%)	2.98	2.35	0.78

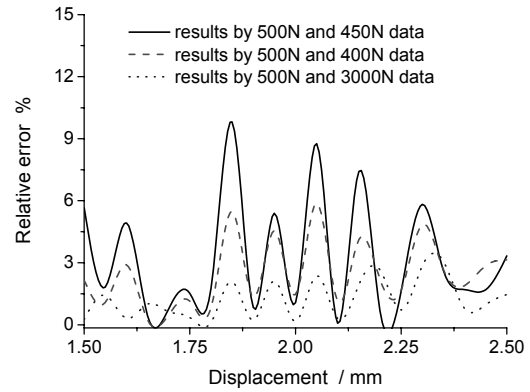


Fig. 9. Relative error of creep exponent obtained from different load levels.

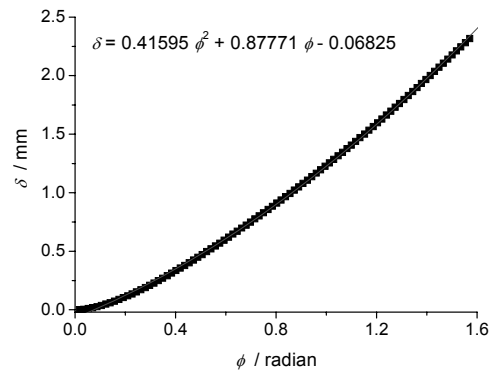


Fig. 10. Relationship between δ and ϕ .

lished. From Fig. 3, it can be seen that the stress distribution inside the specimen is non-uniform. The creep strain also changes inside the specimen. Fortunately, since the specimen is a very thin, the deformation of the disc can be assumed to take place under membrane stress state [6].

Letting φ change from 0° to 90° inside Eq. (11), the δ value can be calculated and the relationship between these two variables is illustrated in Fig. 10. A polynomial expression is regressed as

$$\delta = 0.41595\varphi^2 + 0.87771\varphi - 0.06825 \tag{12}$$

Eliminating the variable φ in Eq. (9) and Eq. (10) yields another implicit function between the ε and δ . With the same procedure, a polynomial expression can be achieved as

$$\varepsilon = 0.31204\delta^2 + 0.4115\delta - 0.00878 \tag{13}$$

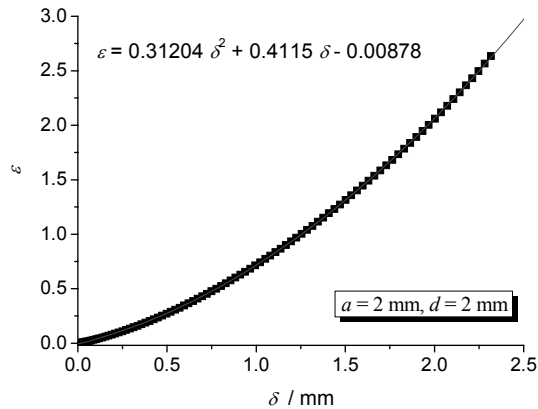


Fig. 11. Relationship between ε and δ .

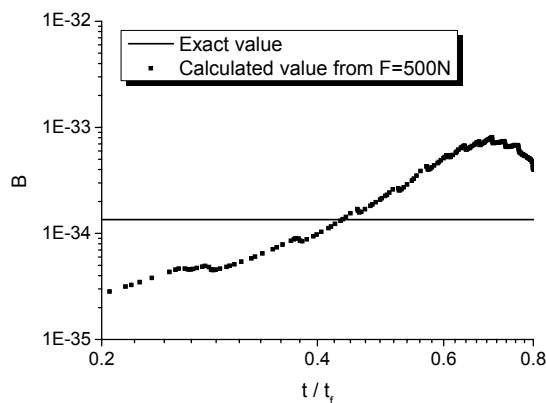


Fig. 12. Creep coefficient obtained during the elapsed time of 0.2~0.8 t/t_f for $F_{sp}=500N$.

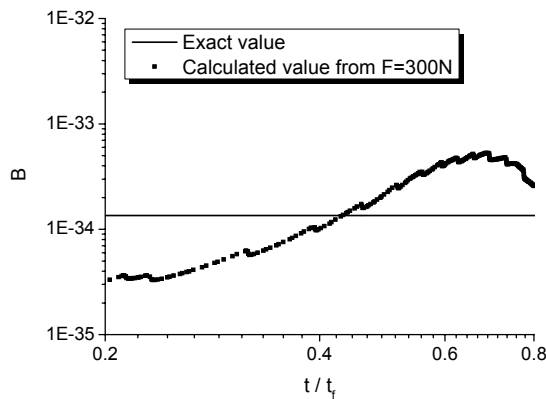


Fig. 13. Creep coefficient obtained during the elapsed time of 0.2~0.8 t/t_f for $F_{sp}=300N$.

And the variation of the ε against δ can be seen in Fig. 11.

Inserting Eq. (13) and Eq. (11) into Eq. (1) will lead to a formula that contains the creep coefficient (B), small punch displacement (δ), displacement rate ($\dot{\delta}$) and the creep exponent (n). Because the latter three variables were obtained in the previous discussion, the creep coefficient can be calculated directly. Two load levels are considered here: $F_{sp} = 500N$ and $F_{sp} = 300N$. The calculated creep coefficients under

these two loads are shown in Fig. 12 and Fig. 13, respectively. Obviously, the variation range of the creep coefficient, B , is much larger than that of the creep exponent, n . However, considering the high dispersal of the creep test data, it is still promising outcome by using the small punch test data to obtain the conventional creep properties. Especially, when $t/t_f = 0.4 \sim 0.5$, the predicted creep coefficient is very close to the exact value. Moreover, the average creep coefficient obtained from 0.2 life time to 0.8 life time has about 30% relative error from the real value, which is rather acceptable from an engineering viewpoint. As mentioned, the selected life time bound (0.2 – 0.8) covers some data that do not belong to steady state creep. If we reduce the life time bound intentionally, it is expected that a more accurate creep coefficient can be obtained. However, it is not easy to determine whether a data boundary pertains the secondary creep state or not. However, from this study, it seems that the data near 0.4 to 0.5 life time is an appropriate obtaining a more accurate creep coefficient of the steady state creep.

5. Conclusions

With the development of the small punch testing technique, it can be used to evaluate a material's creep property at high temperature. However, due to the complicated creep stress distribution and the creep strain evolution in the specimen, a general and effective method to link the small punch creep test data to the conventional tensile creep test data has yet to be found. In this paper, a complete analysis routine is presented to obtain the creep properties by using small punch creep test data. Conclusions are drawn as follows:

- (1) Creep exponent n can be obtained from the small punch creep test directly. The value is expected to be identical to the value obtained from the conventional creep tensile test.
- (2) In the small punch creep test, a more accurate creep coefficient value can be obtained from two large different load levels.
- (3) All the average creep exponent values from our calculation procedure are less than 3%, which shows the high accuracy of our analysis routine proposed in this study.
- (4) The obtained creep coefficient may have a larger error than the creep exponent from the small punch test data. However, the error is still acceptable from an engineering viewpoint, especially when it is obtained from the creep data at $t/t_f = 0.4 \sim 0.5$.

Acknowledgment

The authors are grateful for the financial support by National Research Foundation in Korea (Grant No. 2009-0076175) and Shanghai Natural Science Fund (No. 09ZR1408000).

References

- [1] M. P. Manahan, A. S. Argon and O. K. Harling, The devel-

- opment of a miniaturized disk bend test for the determination of post-irradiation mechanical properties, *Journal of Nuclear Materials*. 104 (1981) 1545-1550.
- [2] J. H. Bulloch, The small punch toughness test: some detailed fractographic information. *International Journal of Pressure Vessels and Piping*. 63 (1995) 177-194.
- [3] A. A. Becker, T. H. Hyde and L. Xia, Numerical analysis of creep in components. *The Journal of Strain Analysis for Engineering Design*. 29 (1994) 185-192.
- [4] D. P. Butt, D. A. Korzekwa and S. A. Maloy, etc. Impression creep behavior of SiC particle-MoSi₂ composites. *Journal of Materials Research*. 11 (1996) 1528-1536.
- [5] T. H. Hyde, K. A. Yehia and A. A. Becker, Interpretation of impression creep data using a reference stress approach. *International Journal of Mechanical Sciences*. 35 (6) (1993) 451-462.
- [6] Z. Yang and Z. W. Wang, Relationship between strain and central deflection in small punch creep specimens. *International Journal of Pressure Vessels and Piping*. 80 (2003) 397-404.
- [7] K. B. Yoon, T. G. Park and S. H. Shim etc. Assessment of creep properties of 9Cr steel using small punch creep testing, *Transaction of KSME(A)* (in Korean). 25 (2001) 1493-1500.
- [8] T. G. Park, S. H. Shim and K. B. Yoon etc. A study on parameters measured during small punch creep testing. *Transaction of KSME(A)* (in Korean). 26 (2002) 171-178.
- [9] K. Milička and F. Dobeš, Small punch testing of P91 steel. *International Journal of Pressure Vessels and Piping*. 83 (9) (2006) 625-634.
- [10] R. Hurst, V. Bicego and J. Foulds, Small punch testing for creep – progress in Europe. *Eighth International Conference on Creep and Fatigue at Elevated Temperatures*. San Antonio, Texas. (2007) 1-6.
- [11] G. I. Barenblatt, Scaling, Self-Similarity, and Intermediate Asymptotics. Cambridge University Press, Cambridge. (1996).
- [12] Y. T. Cheng and C. M. Cheng, What is indentation hardness? *Surface and Coatings Technology*. 133-134 (2000) 417-424.
- [13] Y. J. Liu, B. Zhao, B. X. Xu and Z. F. Yue, Experimental and numerical study of the method to determine the creep parameters from the indentation creep testing. *Materials Science and Engineering A*. 456 (1-2) (2007) 103-108.
- [14] Y. Cao, Determination of the creep exponent of a power-law creep solid using indentation tests. *Mechanics of Time-Dependent Materials*. 11 (2) (2007) 159-172.

- [15] J. Chakrabarty, A theory of stretch forming over hemispherical punch heads. *International Journal of Mechanical Sciences*. 12 (1970) 315-325.
- [16] ABAQUS standard User's Manual, Version 6.5, USA ABAQUS Inc. (2005).



Jian Jun Chen received his Ph. D. degree in Mechanical Engineering from East China University of Science and Technology, China, in 2008. Dr. Chen is currently a Post Doctor at the School of Mechanical Engineering of Chung-Ang University in Seoul, Korea. His research interests are mainly in the material's

high temperature behavior, the creep properties obtained by the non-standard specimen and the interaction between creep and fatigue.



Young Wha Ma received his Ph. D. degree in Mechanical Engineering from Chung-Ang University, Korea in 2007. After that, he worked at Georgia Institute of Technology, U.S.A. as a postdoc. Dr. Ma is currently a research professor at Mechanical Engineering of Chung-Ang University in Seoul, Korea. His interest

is mainly in application of high temperature fracture mechanics to residual life assessment of structural materials including anisotropic materials such as gas turbine blade.



Kee Bong Yoon has conducted various researches in the field of high temperature fracture mechanics including creep and creep-fatigue crack growth behavior of structural materials. He has further interest in application such as residual life assessment of power plant components and risk based maintenance approaches.

After 10 year experiences in a national research institute he moved to the mechanical engineering department in Chung Ang University. He received his B.E. and M.S from Seoul National University and KAIST respectively and he got Ph.D. degree from Georgia Tech in U.S.A. Currently he is the research dean of the University.